Összegzés

$f : \mathbb{Z} \to \mathbb{Z}$, $[m..n] \subset \mathbb{Z}$

$$A = \underset{m}{\mathbb{Z}} \times \underset{n}{\mathbb{Z}} \times \underset{s}{\mathbb{Z}} (\times \underset{k}{\mathbb{Z}})$$
$$B = \underset{m'}{\mathbb{Z}} \times \underset{n'}{\mathbb{Z}}$$
$$Q = (m = m' \wedge n = n' \wedge m \leq n+1)$$
$$R = (Q \wedge s = \sum_{i=m}^{n} f(i))$$

$k, s := m-1, 0$

$k \neq n$

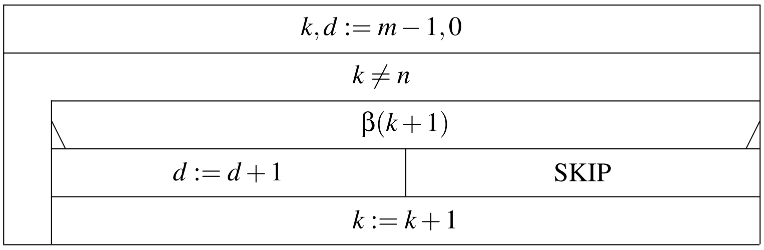
  $s := s + f(k+1)$

  $k := k+1$

Számlálás

$\beta : \mathbb{Z} \to \mathbb{L}$, $[m..n] \subset \mathbb{Z}$

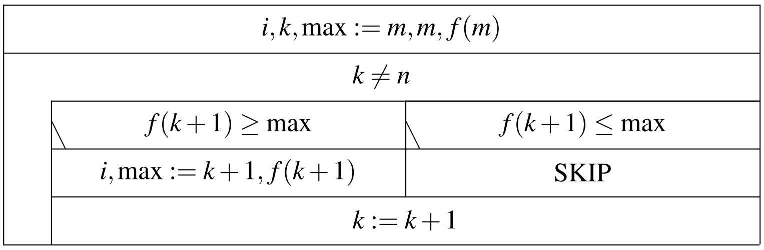
$A = \underset{m}{\mathbb{Z}} \times \underset{n}{\mathbb{Z}} \times \underset{d}{\mathbb{N}_0} (\times \underset{k}{\mathbb{Z}})$
$B = \underset{m'}{\mathbb{Z}} \times \underset{n'}{\mathbb{Z}}$
$Q = (m = m' \wedge n = n' \wedge m \leq n+1)$
$R = (Q \wedge d = \sum_{i=m}^{n} \chi(\beta(i)))$



Maximumkeresés

$\mathcal{H}$ rendezett halmaz és $f : \mathbb{Z} \to \mathcal{H}$, $[m..n] \subset \mathbb{Z}$

$A = \underset{m}{\mathbb{Z}} \times \underset{n}{\mathbb{Z}} \times \underset{i}{\mathbb{Z}} \times \underset{\max}{\mathcal{H}} (\times \underset{k}{\mathbb{Z}})$
$B = \underset{m'}{\mathbb{Z}} \times \underset{n'}{\mathbb{Z}}$
$Q = (m = m' \wedge n = n' \wedge m \leq n)$
$R = (Q \wedge i \in [m..n] \wedge \max = f(i) \wedge \forall j \in [m..n] : f(j) \leq f(i))$



Alteres általánosításainak esetei:

- Ha a max nincs a feladat állapotterében:
  $R_{\text{tétel}} = (Q \wedge i \in [m..n] \wedge \max = f(i) \wedge \forall j \in [m..n] : f(j) \leq f(i))$
  $\Downarrow$
  $R = (Q \wedge i \in [m..n] \wedge \forall j \in [m..n] : f(j) \leq f(i))$
- Ha az i nincs a feladat állapotterében:
  $R_{\text{tétel}} = (Q \wedge i \in [m..n] \wedge \max = f(i) \wedge \forall j \in [m..n] : f(j) \leq f(i))$
  $\Downarrow$
  $R = (Q \wedge \exists i \in [m..n] : (\max = f(i) \wedge \forall j \in [m..n] : f(j) \leq f(i)))$

Feltételes maximumkeresés

$\mathcal{H}$ rendezett halmaz és $f : \mathbb{Z} \to \mathcal{H}$, $\beta : \mathbb{Z} \to \mathbb{L}$, $[m..n] \subset \mathbb{Z}$

$A = \underset{m}{\mathbb{Z}} \times \underset{n}{\mathbb{Z}} \times \underset{i}{\mathbb{Z}} \times \underset{\max}{\mathcal{H}} \times \underset{l}{\mathbb{L}} (\times \underset{k}{\mathbb{Z}})$
$B = \underset{m'}{\mathbb{Z}} \times \underset{n'}{\mathbb{Z}}$
$Q = (m = m' \land n = n' \land m \leq n+1)$
$R = (Q \land l = (\exists i \in [m..n] : \beta(i)) \land l \to (i \in [m..n] \land \beta(i) \land \max = f(i) \land \forall j \in [m..n] : \beta(j) \to (f(j) \leq f(i))))$

<table>
<tr><td colspan="5">$k, l := m-1, \text{hamis}$</td></tr>
<tr><td colspan="5">$k \neq n$</td></tr>
<tr><td rowspan="4"></td><td>$\neg\beta(k+1)$</td><td>$\beta(k+1) \land \neg l$</td><td colspan="2">$\beta(k+1) \land l$</td></tr>
<tr><td rowspan="2">SKIP</td><td rowspan="2">$l, i, \max := \text{igaz}, k+1, f(k+1)$</td><td>$f(k+1) \geq \max$</td><td>$f(k+1) \leq \max$</td></tr>
<tr><td>$i, \max := k+1, f(k+1)$</td><td>SKIP</td></tr>
<tr><td colspan="4">$k := k+1$</td></tr>
</table>

Alteres általánosításainak esetei:

- Ha a max nincs a feladat állapotterében:
$$R_{\text{tétel}} = (Q \land l = (\exists i \in [m..n] : \beta(i)) \land l \to (i \in [m..n] \land \beta(i) \land \max = f(i) \land \forall j \in [m..n] : \beta(j) \to (f(j) \leq f(i))))$$
$$\Downarrow$$
$$R = (Q \land l = (\exists i \in [m..n] : \beta(i)) \land l \to (i \in [m..n] \land \beta(i) \land \forall j \in [m..n] : \beta(j) \to (f(j) \leq f(i))))$$

- Ha az i nincs a feladat állapotterében:
$$R_{\text{tétel}} = (Q \land l = (\exists i \in [m..n] : \beta(i)) \land l \to (i \in [m..n] \land \beta(i) \land \max = f(i) \land \forall j \in [m..n] : \beta(j) \to (f(j) \leq f(i))))$$
$$\Downarrow$$
$$R = (Q \land l = (\exists i \in [m..n] : \beta(i)) \land l \to (\exists i \in [m..n] : (\beta(i) \land \max = f(i) \land \forall j \in [m..n] : \beta(j) \to (f(j) \leq f(i)))))$$

Lineáris keresés 1.0

$\beta : \mathbb{Z} \to \mathbb{L}$

$A = \underset{m}{\mathbb{Z}} \times \underset{i}{\mathbb{Z}}$

$B = \underset{m'}{\mathbb{Z}}$

$Q = (m = m' \wedge \exists j \geq m : \beta(j))$

$R = (Q \wedge i \geq m \wedge \beta(i) \wedge \forall j \in [m..i-1] : \neg\beta(j))$

$i := m$
$\neg\beta(i)$
$i := i+1$

Lineáris keresés 2.0

$A = \underset{m}{\mathbb{Z}} \times \underset{i}{\mathbb{Z}} (\times \underset{l}{\mathbb{L}})$

$i, l := m-1, \text{hamis}$
$\neg l$
$l := \beta(i+1)$
$i := i+1$

Lineáris keresés 3.0: $\beta = \gamma \vee \delta$

$A = \underset{m}{\mathbb{Z}} \times \underset{i}{\mathbb{Z}} \times \underset{u}{\mathbb{L}} (\times \underset{v}{\mathbb{L}})$

$Q = (m = m' \wedge \exists j \geq m : \beta(j))$

$R = (Q \wedge u = (\exists j \geq m : \gamma(j) \wedge \forall k \in [m..j-1] : \neg\delta(k)) \wedge u \to (i \geq m \wedge \gamma(i) \wedge \forall j \in [m..i-1] : \neg\beta(j)))$

$i, u, v := m-1, \text{hamis}, \text{hamis}$
$\neg u \wedge \neg v$
$u, v := \gamma(i+1), \delta(i+1)$
$i := i+1$

Lineáris keresés 2.8: Lin. ker. 3.0, de $\delta =$ nem értük el n-et

$A = \underset{m}{\mathbb{Z}} \times \underset{n}{\mathbb{Z}} \times \underset{i}{\mathbb{Z}} \times \underset{l}{\mathbb{L}}$

$B = \underset{m'}{\mathbb{Z}} \times \underset{n'}{\mathbb{Z}}$

$Q = (m = m' \land n = n' \land m \leq n+1)$

$R = (Q \land l = (\exists j \in [m..n] : \beta(j)) \land l \rightarrow (i \in [m..n] \land \beta(i) \land \forall j \in [m..i-1] : \neg \beta(j)))$

$i, l := m-1, \text{hamis}$	
$\neg l \land i \neq n$	
	$l := \beta(i+1)$
	$i := i+1$

Általánosításának tipikus esetei:

- Ha az i nincs a feladat állapotterében (alteres):

$$\begin{aligned} R_{\text{tétel}} &= (Q \land l = (\exists j \in [m..n] : \beta(j)) \land l \rightarrow (i \in [m..n] \land \beta(i) \land \forall j \in [m..i-1] : \neg \beta(j))) \\ &\Downarrow \\ R &= (Q \land l = (\exists j \in [m..n] : \beta(j)) \land l \rightarrow (\exists i \in [m..n] : \beta(i) \land \forall j \in [m..i-1] : \neg \beta(j))) \\ &= (Q \land l = (\exists j \in [m..n] : \beta(j))) \end{aligned}$$

- Ha nem követeljük meg, hogy az első megfelelő értéket találjuk meg:

$$\begin{aligned} R_{\text{tétel}} &= (Q \land l = (\exists j \in [m..n] : \beta(j)) \land l \rightarrow (i \in [m..n] \land \beta(i) \land \forall j \in [m..i-1] : \neg \beta(j))) \\ &\Downarrow \\ R &= (Q \land l = (\exists j \in [m..n] : \beta(j)) \land l \rightarrow (i \in [m..n] \land \beta(i))) \end{aligned}$$

Logaritmikus keresés: $\mathcal{H}$ halmazon van egy rendezési reláció és $f : \mathbb{Z} \to \mathcal{H}$ monoton növekedő., $[m..n] \subset \mathbb{Z}$.

$A = \underset{m}{\mathbb{Z}} \times \underset{n}{\mathbb{Z}} \times \underset{h}{\mathcal{H}} \times \underset{i}{\mathbb{Z}} \times \underset{l}{\mathbb{L}} (\times \underset{u}{\mathbb{Z}} \times \underset{v}{\mathbb{Z}})$
$B = \underset{m'}{\mathbb{Z}} \times \underset{n'}{\mathbb{Z}} \times \underset{h'}{\mathcal{H}}$
$Q = (m = m' \wedge n = n' \wedge h = h')$
$R = (Q \wedge l = (\exists j \in [m..n] : f(j) = h) \wedge l \to (i \in [m..n] \wedge f(i) = h))$

<table>
<tr><td colspan="4">$u, v, l := m, n, \text{hamis}$</td></tr>
<tr><td rowspan="4"></td><td colspan="3">$\neg l \wedge u \leq v$</td></tr>
<tr><td colspan="3">$i := \lceil (u + v)/2 \rceil$</td></tr>
<tr><td>$f(i) < h$</td><td>$f(i) = h$</td><td>$f(i) > h$</td></tr>
<tr><td>$u := i + 1$</td><td>$l := \text{igaz}$</td><td>$v := i - 1$</td></tr>
</table>